NEW METHODS OF OPTIMIZATION SHAPE FOR MAXIMAL DRAG OR LIFT FORCE AIRFOILS

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***Abstract**: Direct and inverse boundary value problems are solved and the solution of an optimization shape problem is obtained analytically in the case of some nonlinear integral functionals. The plain potential flow of an inviscid fluid is considered in the absence of mass force (Hyp). The flow – unlimited jet – encounters a symmetrical curvilinear obstacle (the Helmholtz scheme). For invers problems there are derived singular integral equations and the movement is obtained in the auxiliary canonical half plane. Next, a new method of optimization problem is solved analytically. The design of the optimal airfoil is performed. The drag/lift coefficient, nonlinear integral functionals and other geometrical parameters are computed in the case of a given distribution of the velocity or angle on the profile/airfoil. The main applications of this contributions are related to the optimization of leading edges, modeling special airfoils for different categories of low speed small UAVs, but also in determining efficient systems for recovery of light UAVs (deflectors, special braking parachute, determal systems).*

***Mathematics Subject Classifications 2010**: 76B07, 49Q10, 35J25, 35F15, 45E05.*

***Keywords**: small UAVs, optimal airfoils, systems for recovery.*

1. INTRODUCTION

The aerodynamic performances are given by a nonlinear functional [1,2]. We use the Maklakov-Jensen inequality and the inverse methods for singular integral equations, [3,4]. The velocity field in the physical domain w(z) = u(x; y) + iv(x; y).

The complex potential f(z) and the complex velocity w(z) are defined:

f(z) = φ(x, y) + i ψ(x; y);

$$\overline{w} = u - iv = \frac{df}{dz} = Ve^{-i\theta} \quad (1)$$

with $\varphi(x,y)$, $i\psi(x,y)$ the velocity potential and the stream function. The velocity is:

$V = (u^2 + v^2)^{\frac{1}{2}}$ and $\theta = \arg \overline{w}$

Let D_ζ, $\zeta = \xi + i\eta, \eta \geq 0$ a canonical auxiliary domain which corresponds to the plane $D_z, y \geq 0$ and $D_f, \psi \geq 0$. The aim is to determine $f = f(\zeta), D_f \leftrightarrow D_\zeta$, with $f_{\overline{\zeta}} = 0$. Let the Jukovski function $\omega = \omega(\zeta)$.

Along the free lines $V = V^0$, we have:

$$\omega = t + i\theta,\ \overline{w} = V^0 e^{-\omega},\ t = \ln\frac{V^0}{V},$$

$$0 \leq V \leq V^0,\ w_{\overline{\zeta}} = 0 \quad (2)$$

$f = f(\zeta)$, $\omega = \omega(\zeta)$ and $z = z(\zeta)$ corresponds to the conformal mappings $D_f^+ \leftrightarrow D_\zeta^+$, $D_\omega^+ \leftrightarrow D_\zeta^+$, $D_z^+ \leftrightarrow D_\zeta^+$, and it is found $f = f(\zeta) = A\zeta$. In addition: $\eta = 0, \psi = 0$ and $\frac{\delta\varphi}{\delta\eta}|_{\eta=0} = 0$. The boundaries of D_z, D_f correspond to the boundary of $D_\zeta, \eta = 0$, $\xi \in (-\infty, \infty)$, on which we have t $\psi = \text{const.}$ On $\eta = 0$ we have:

$$z(\xi) = \int \varphi'_\xi \frac{e^{i\theta}}{V} d\xi,\quad V = V(\xi),\ \theta = \theta(\xi) \quad (3)$$

Using $\omega(\zeta)$ and (3) we find the ecuation of the obstacle and the freelines. The flow encounters a curvilinear symmetrical obstacle

(BOB'), in the points B, B' the free streamlines $(BC),(B'C')$ are detached; $\vec{V}(C)=\vec{V}(C')=V^0\,\vec{i}$ (*downstream*). Between D_z^+ and D_ζ^+, we suppose that the boundary (A_0OBC) corresponds to $\eta=0,\xi\in(-\infty,\infty)$. The obstacle (OB) is the segment (-1,1) (Fig. 2) and the length of (OB) in D_z is L.

The integral equations. The aim is to find $\omega=\omega(\zeta)$ defined in $D\zeta$ analytically in two cases: (1) if $\omega(\xi)=t(\xi)+i\theta(\xi)$ is known on $\eta=0:\theta=0,\xi\in(-\infty,-1);\theta=\theta(\xi)$ or (2) if it is given $t=t(\xi),\xi\in(-1,1),t=0,\xi\in(1,0)$. These mixed problems have the solutions (Dirichlet, Volterra, Riemann-Hilbert [1]):

$$\omega(\zeta)=\frac{\sqrt{\zeta+1}}{\pi i}\int_{-1}^{1}\frac{t(s)}{\sqrt{s+1}}\frac{ds}{s-\zeta},$$

$$\omega(\zeta)=\frac{\sqrt{\zeta-1}}{\pi}\int_{-1}^{1}\frac{\theta(s)}{\sqrt{1-s}}\frac{ds}{s-\zeta},$$

$$\lim_{\zeta\to\infty}\omega(\zeta)=0,\quad \zeta\in D_\zeta^+ \tag{4}$$

Applying the Sohotski-Plemelj relation [5], it results the following singular integral equations:

$$\omega=t+i\theta,$$

$$\theta(\xi)=-\frac{\sqrt{\xi+1}}{\pi i}\int_{-1}^{1}\frac{t(s)}{\sqrt{s+1}}\frac{ds}{s-\xi},$$

$$t(\xi)=\frac{\sqrt{1-\xi}}{\pi}\int_{-1}^{1}\frac{\theta(s)}{\sqrt{1-s}}\frac{ds}{s-\xi},\xi\in(-1,1) \tag{5}$$

The practical importance of these inverse problems is that if it is known *a priori* the distribution of the velocity, or of the pressure, or of the angle, on the profile then the shape of the profile may be computed *a posteriori*. Finally the pressure, the drag coefficient and the length of the profile are respectivelly:

$$P=\frac{\rho V^{o^2}L}{2}I[t],\quad I[t]=\frac{\left(\int_{-1}^{1}\frac{t(s)}{\sqrt{s+1}}ds\right)^2}{\pi\int_{-1}^{1}e^{t(s)}ds} \tag{6}$$

$$C_x=\frac{2P}{\rho V^{o^2}L}=I[t],\ L=A\int_{-1}^{1}\frac{ds}{V(s)}=\frac{A}{V^o}\int_{-1}^{1}e^{t(s)}ds$$

We obtain the profile with the distribution of $V(\xi),t(\xi)\,\text{or}\,\theta(\xi)$ linked to a parameter selected in order to extremize the aerodynamic drag. The distribution of the velocity on the profile must satisfy the Brillouin-Villat (B-V) conditions:

$$V(0)=V(\xi=-1)=0,\ \ V(B)=V(\xi=1)=V^0$$
$$V'(\xi)>0,\ \xi\in(-1,1) \tag{7}$$

2. BASIC ASPECTS OF DIFFERENT SIMPLE AERODYNAMIC SHAPE OPTIMIZATION

Let the curvilinear axis-symmetric profiles, with a given speed distribution (Fig. 1).

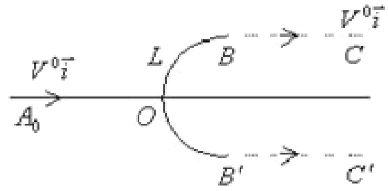

Fig. 1 Curvilinear axis-symmetric profiles

In the inverse problem, let the following distribution of the velocity on the obstacle:

$$V=V(\xi)=V^0\sqrt{\frac{1+\xi}{2}},\quad t=t(\xi)=\ln\frac{V^0}{V}=\ln\sqrt{\frac{2}{1+\xi}},$$
$$\xi\in(-1,1) \tag{8}$$

Where:

$$V(O)=V(\xi=-1)=0,\quad V(B)=V(\xi=1)=V^0$$

This distribution is motivated by the fact that the function $V=V(\xi)$ must satisfy the condition $V(-1)=0$ and the convergence of the integrals. In this case:

$$V(\xi)=(1+\xi)^\alpha.h(\xi),\ 0\prec\alpha\prec1,\ h(-1)\neq0.$$

Thus, (8) is a choice with $\alpha=1/2$ and $V'(\xi)\succ0$. From (8) and (5) we obtain the velocity angle along (OB):

$$\theta(\xi)=\frac{\pi}{2}-T\left(\sqrt{\frac{1+\xi}{2}}\right),\xi\in(-1,1) \tag{9}$$

Here,

$$T(a)=\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{a^{2n+1}}{(2n+1)^2}=\frac{1}{\pi}\left[Li_2(a)-Li_2(-a)\right]$$

$$Li_2(a)=\sum_{n=1}^{\infty}\frac{a^n}{n^2},\qquad |a\prec1| \tag{10}$$

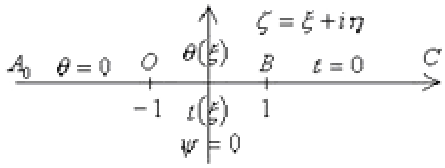

Fig. 2 Obstacle OB and free line BC

It's easy to see that:

$$\frac{dT}{da} = \frac{1}{\pi a}\ln\frac{1+a}{1-a}, \quad T(a) = \frac{1}{\pi}\int_0^a \ln\frac{1+a}{1-a}\frac{da}{a},$$

$$T(\pm 1) = \pm\frac{\pi}{4}, \quad T(0) = 0$$

From (9) we have $\theta(O) = \theta(\xi = -1) = \frac{\pi}{2}$, $\theta(B) = \theta(\xi = 1) = \frac{\pi}{4}$ and we observe that the curve (BOB') has a continuous tangent with $\theta'(\xi) \prec 0$ which assure the downstream convexity. We remark that (8) and (9) are inversion formulae for the integral singular equations (5), and we have in the hodograph plane (V,θ) on the profile:

$$\theta(V) = \frac{\pi}{2} - T\left(\frac{V}{V^0}\right), \quad 0 \le V \le V^0,$$

$$\theta = \theta(V), \quad V = V(\theta) \tag{11}$$

We consider the general case:

$$V = V(\xi) = V^0\left(\frac{1+\xi}{2}\right)^{\alpha}, \quad \alpha \in (0,1),$$

$$t = t(\xi) = \ln\frac{V^0}{V} = \left(\frac{2}{1+\xi}\right)^{\alpha}, \quad \xi \in (-1,1) \tag{12}$$

From (5) and (12) we obtain the velocity angle along (OB):

$$\theta(\xi) = \alpha\pi - 2\alpha T\left[\sqrt{\frac{1+\xi}{2}}\right]; \xi \in [-1,1] \tag{13}$$

From (13) we have:

$$\theta(0) = \theta(\xi = -1) = \alpha\pi; \quad \theta(B) = \theta(\xi = 1) = \frac{\alpha\pi}{2}$$

Knowing L and V^0 from (6) and (13) it is possible to determine the parameter A:

$$L = \frac{A}{V^0}\int_{-1}^{1} e^t d\xi = \frac{A}{V^0}\frac{2}{1-\alpha}, \alpha \in \left[0, \frac{1}{2}\right] \tag{14}$$

It results the equations of the obstacle (OB):

$$X(\xi) = \frac{x(\xi)}{L} = \frac{\sqrt{2}}{4}\int_{-1}^{\xi}\frac{\cos\theta(s)}{\sqrt{1+s}}ds,$$

$$Y(\xi) = \frac{y(\xi)}{L} = \frac{\sqrt{2}}{4}\int_{-1}^{\xi}\frac{\sin\theta(s)}{\sqrt{1+s}}ds, \quad \xi \in (-1,1) \tag{15}$$

One similarly obtains the equations of the free line (BC). Next, we compute the resultant of pressures for the whole profile (BOB') and the drag coefficient C_x (6):

$$P = \frac{\rho(V^0)^2 L}{2}\frac{16\alpha^2(1-\alpha)}{\pi} \tag{16}$$

The case in which we have just one parameter with $C_x(\alpha) \Rightarrow C_x(\alpha)$ is maximal.

$$C_x = \frac{P}{\frac{\rho(V^0)^2 L}{2}} = \frac{16\alpha^2(1-\alpha)}{\pi}$$

$$C'_x(\alpha) \equiv 0, \; C'(\alpha) > 0 \tag{17}$$

If we take $\alpha \in [0, \frac{1}{2}]$ the $C_{max} = C_x(\alpha = \frac{1}{2}) = \frac{2}{\pi} = 0{,}638$ (edges concave, Fig. 3) and for $\alpha \in (0,1)$ we obtaining (edges accolade, Fig. 4):

$$C_{max} = C_x(\alpha = \frac{2}{3}) = \frac{64}{27\pi} = 0{,}75\,.$$

In Fig. 3 and Fig. 4 we present the profiles for maximal drag.

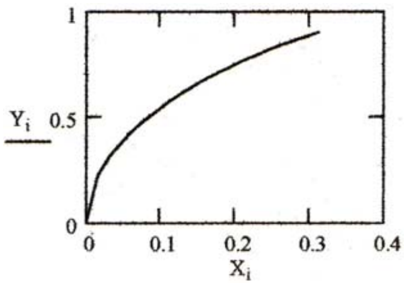

Fig. 3 Profile with concave edges

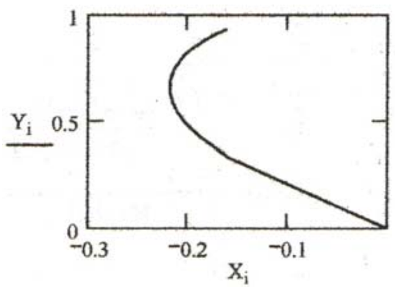

Fig. 4 Profile with accolade edges

The general optimization problem of shape of the wing with subsonic leading edge (sail prove):

A) Profiles with velocity distribution $V(\xi)$, (8) (12), [8].

B) Profiles with angles distribution $\theta(\xi)$**, plate or ogival profiles.** In this case distribution angle $\theta = \theta(\xi)$ from ogival profiles:

$$\theta(\xi) = \alpha\pi + \pi(\gamma - \alpha)\sqrt{\frac{1+\xi}{2}},$$

$$\theta(O) = \theta(\xi = -1) = \alpha\pi,$$

$$\theta(B) = \theta(\xi = 1) = \gamma\pi,\ \gamma < \alpha \le \frac{1}{2} \tag{18}$$

Replacing $\theta(\xi)$ in the integral equations (5) obtained distribution velocity on the ogival profile with:

$$V(O) = 0,\ V(B) = V^0,$$

$$t = \ln\frac{V^0}{V} = \pi(\gamma - \alpha)\sqrt{\frac{1-\xi}{2}} + \ln\frac{\sqrt{2}+\sqrt{1-\xi}}{\sqrt{2}-\sqrt{1-\xi}},$$

$$V = V^0 \tag{19}$$

Calculating the pressure and C_x (6), we obtain:

$$P = \frac{\rho V^0 A\pi}{4}(4\alpha - \delta)^2,$$

$$U(\xi) = \sqrt{\frac{1-\xi}{2}}, \delta = \pi(\alpha - \gamma),$$

$$C_x(\delta) = \frac{1}{2}\frac{(4\alpha-\delta)^2}{\int_{-1}^{1} e^{-\delta U}\left(\frac{1+U}{1-U}\right)^2 d\xi} \tag{20}$$

a) Considering $\alpha = \gamma = \frac{1}{2}$, $\theta = \frac{\pi}{2}$, $t = \frac{1}{2}\ln\frac{1+U}{1-U}$ and $V = V^0\sqrt{\frac{1-U}{1+U}}$, we have "Helmholtz plate" and

$$C_x^H = \frac{2\pi}{4+\pi} \approx 0{,}87980 \tag{21}$$

b) Considering $\alpha = \gamma < \frac{1}{2}, \theta = \alpha\pi < \frac{\pi}{2}$ it is a triangle profile for the Newton problem (i.e. to minimal drag ogival profiles):

$$C_x^*(\alpha) = \frac{4\pi\alpha^2}{\left[1 + 2\alpha + 4\alpha^2 \cdot \beta(1-\alpha)\right]\sin\alpha\pi} \le C_x^H,$$

$$\beta(x) = \int_0^1 \frac{t^{x-1}}{1+\lambda}d\lambda \tag{22}$$

Particularly, $\alpha = \frac{1}{2}, \beta(\frac{1}{2}) = \frac{\pi}{2}, C_x^*\left(\frac{1}{2}\right) = C_x^H$.

For polygonal profiles, the integral equation method is more efficient than the hodographic method from the theory of jets.

C) Parametric profiles. The best deflector

We consider profiles with the distributions:

$$V = V^0 e^{-\delta\sqrt{\frac{1-\xi}{2}}}\sqrt{\frac{1+\xi}{2}},$$

$$t_0(\xi) = \delta\sqrt{\frac{1-\xi}{2}} - \ln\sqrt{\frac{1+\xi}{2}},\ \xi \in (-1,1),$$

satisfying the (B-V) conditions. Using $t_0(\xi)$ within (5), it results:

$$\theta(\xi) = \frac{\pi}{2} + \delta\sqrt{\frac{1+\xi}{2}} - T(\sqrt{\frac{1+\xi}{2}}),$$

where: $T(\alpha) = \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{\alpha^{2n+1}}{(2n+1)^2} = \frac{1}{\pi}\int_0^{\alpha}\ln\frac{1+s}{1-s}\frac{ds}{s}$,

$$T(\pm 1) = \frac{\pi}{4},\ T(0) = 0.$$

For $\delta \in [0, \frac{2}{\pi}]$, the profile is prove type, while for $\delta \in (\frac{2}{\pi}, \frac{3\pi}{2})$ the profile is deflector. With (6), it results:

$C_x = I(t) = \dfrac{(\delta\pi + 4)^2}{8\pi\int_0^{\frac{\pi}{2}} e^{\delta\sin\theta}\sin\theta d\theta}$, $C_x(\delta)$ is increasing and consequently

$$C_x^P(\delta = 0) = \frac{2}{\pi} \approx 0.638, C_x^M(\delta = \frac{2}{\pi}) \approx 0.86053.$$

For $\delta > \frac{2}{\pi}$ we find the maximal resistance deflector. The main tool is the Jensen inequality: if $f(x) \ge 0$ and $g(x)$ are integrable functions in α^{2n+1}, then

$$\int_a^b f(x)e^{g(x)}dx \ge \int_a^b f(x)dx \exp\frac{\int_a^b f(x)g(x)dx}{\int_a^b f(x)dx}$$

Where the inequality occurs if g is constant $(g \equiv g_0)$, [2,3,4]. The major idea to maximize a functional I[u] is based on the use of Jensen inequality, I[u] < J[u], whose maximal point U_0 is constant.

$$I[u] \le J[u] \le J[U_0] = I[U_o] = \max.$$

For $t(\xi) = t_0(\xi) + u(\xi), \xi \in (-1,1)$, where $u(\xi)$ is disturbed velocity applying the Jensen inequality to the denominator of C_x (6), it results $C_x = I(t(u)) \le J(U(u)) = \dfrac{(U + 2\sqrt{2})^2}{4\pi e^{\frac{\sqrt{2}}{4}U}}$,

with $U = \displaystyle\int_{-1}^{1} \frac{u + \delta\sqrt{\frac{1-\xi}{2}}}{\sqrt{1+\xi}} d\xi$. The maximum value of J(U) (where $J'(U_0) = 0$) is obtained for $U_0 = 2\sqrt{2} = g_0 \displaystyle\int_{-1}^{1} \frac{d\xi}{\sqrt{1+\xi}}$. The equality case of the Jensen's inequality implies that $g_0 = 1,\ u = 1 - \delta\sqrt{\frac{1-\xi}{2}}$ maximizes the functional I(t(u)). Further, we obtain:

$$t = 1 + \ln\sqrt{\frac{2}{1+\xi}},\ V = \frac{V^0}{e}\sqrt{\frac{1+\xi}{2}},$$

$$I_{max} = J(2\sqrt{2}) = \frac{8}{\pi e} = C_x^D \approx 0.936797.$$

The obtained distribution of $V(\xi)$ gives the best deflector of the family $V(\xi,\delta)$. For $\delta = 0$ the impermeable parachute is found again, [3,4]. This result is in agreement with that $(\delta = 0)$ obtained by Maklakov before using Levi-Civita method [2].

For 3D axis-symmetric profiles:

$$C_x^* = \frac{8}{\pi} \frac{(\delta\pi + 2)^2}{\int_0^{\pi} e^{\delta \sin\theta}(\sin\theta + \sin^2\theta) d\theta},$$

$$\delta = \frac{1}{2} - \gamma, C_x^*(\max) \approx C(\delta = 0.410916) \approx 0.911$$

For $\alpha = \gamma = \frac{1}{2},\quad \delta = 0,\quad \theta = \frac{\pi}{2}$, it is obtained the Helmholtz plane plate, with $C_x^H = \frac{2\pi}{\pi + 4} \approx= 0.87980$. It may be observed that $C_x^P < C_x^M < C_x^H < C_x^* < C_x^D$, $A_P = \frac{LV^0}{4}$,

$$A_M = \frac{LV^0}{2(\pi+4)},\ A_H = \frac{LV^0}{\pi+4},$$

$$A^* \approx \frac{LV^0}{4(1 + 0.410916)},\ A_D = \frac{LV^0}{4R}.$$

For the minimal drag case it results: $\theta(\xi) = \alpha = \gamma < \frac{\pi}{2}$ and

$$\min C_x^*(\alpha) = \frac{4\pi\alpha^2}{(1 + 2\alpha + 4\alpha^2\beta(1-\alpha))\sin\alpha\pi} < C_x^H$$

Where $\beta(x) = \displaystyle\int_0^1 \frac{t^{x-1}}{1+t} dt$ (triangle airfoil).

3. THE LIFT MAXIMIZATION

Let a symmetrical curve plate (AB) in a paralel flow with the chord. Let be L(AB) length of (AB) and l length of chord knowns again A_0A, BB_0 free lines with A_0AMBB_0 stream line $\psi = 0$ (Fig. 5, 6).

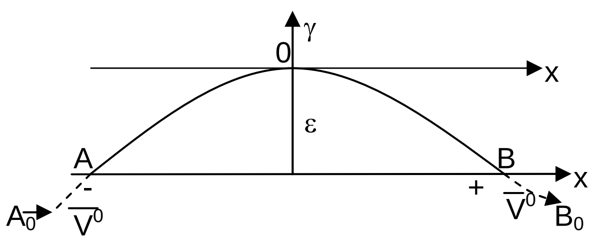

Fig. 5 Symmetrical curve plate (AB)

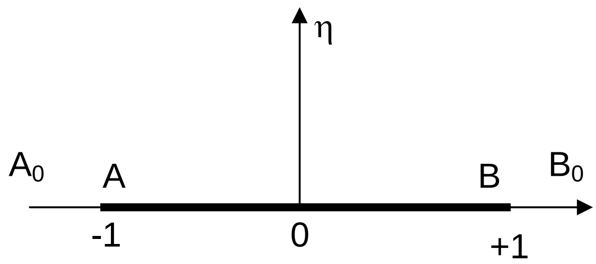

Fig. 6 Free lines A_0A, BB_0

We denote by $k = \frac{L-l}{l}$ and we will must to determine optimal geometrical shape for maximum lift P (rectangular on chord). We consider T1, T2 theorems with integral equations and we will determine potential function $f = f(\zeta)$ and $\overline{\omega} = \overline{\omega}(\xi)$ in the upper half plane, $\eta \geq 0$; the plate (AB) being lateral acting of wind with the speed $V^0\vec{i}$. Let be $f(\zeta)$ complex potential and $\overline{\omega} = \frac{df}{dz} = \frac{df}{d\zeta}\frac{d\zeta}{dz}$,

$f(\zeta) = AV^0\zeta;\ dz = \varphi'_\xi e^{i\theta} d\xi,\ \psi = 0,\ \eta = 0$.

In this case:

$\omega(\zeta) = -\frac{1}{\pi i}\int_{-1}^{1}\frac{t(s)}{s-\zeta}$ and the profile angle:

$$\theta(\xi) = \frac{1}{\pi}\int_{-1}^{1}\frac{t(s)}{s-\xi}ds,\ \xi \in (-1,\ 1).$$

$$z(\xi) = \int_{-1}^{\xi}\varphi_\xi \frac{e^{i\theta}}{V(\xi)}d\xi, dS = \varphi_\xi \frac{d\xi}{V(\xi)} \qquad (24)$$

$$L = AV^0\int_{-1}^{1}\frac{d\xi}{V(\xi)} = A\int_{1}^{1}e^{t(S)}dS \qquad (25)$$

The resultant of pressures is:

$$X + iY = i\rho V^{0^2} A\oint e^{\omega(\zeta)}d\zeta \qquad (26)$$

And because the symmetry,

$$X = 0, Y = \rho V^{0^2} A\int_{-1}^{1}t(S)dS$$

The lift will be:

$$Y = \rho V^{0^2} LJ(t),\ J(t) = \frac{2\int_{-1}^{1}t(S)dS}{\int_{-1}^{1}e^{t(S)}dS}$$

We search the velocity distribution on (AB) by using Jensen's inequality at the denominator of $J \leq I$ so that the functional J(t) to be maximum. $H = \int_{-1}^{1}t(S)dS$ we will obtain $J \leq I = He^{-\frac{H}{2}}$ in the case equal the functional is I_{max}.

For obtaining the maxim, $I'(H) = 0$, with $H \succ 0, I' = e^{-\frac{H}{2}}\left(1 - \frac{H}{2}\right)$.

For $H = 2,\ I_{max} = \frac{2}{e}$ and $t(\xi) = 1$.

In this case:

$$A = \frac{L}{2e},\ V = \frac{V^0}{e},\ \theta(\xi) = \frac{1}{\pi}\int_{-1}^{\xi}\frac{dS}{(S-\xi)} =$$

$$= \frac{1}{\pi}\ln\frac{\sqrt{2}+\sqrt{1+\xi}}{\sqrt{2}-\sqrt{1+\xi}},\ \xi \in (-1,1)$$

And with

$$\frac{1}{L} = \int_{-1}^{1}\cos\theta(\xi)d\xi = \frac{2e}{e^2-1},\ k = sh(e-1).$$

From Y with I_{max} we obtain the lift coefficient:

$$(23)\quad C_y = \frac{Y}{\rho V^{0^2} L} \leq C_{y\,max}, C_{y\,max} = \frac{2}{e}(1+k).$$

The optimal lift for plate will be:

$$P_{max} = C_{y\,max} \cdot S,\ k = 0{,}175,$$

$$C_{y\,max} \approx 0{,}876 \qquad (28)$$

Wu and Whitney have study this problem with application for the case of "para-slope".

4. LIFT MAXIMIZATION FOR THE CASE OF PARAMETRICAL VELOCITY DISTRIBUTION INPUT

It was demonstrate that for obtaining the maximal lift, we analyzed the distribution of speeds on the A_0AOBB_0 line:

$V(\xi) = V^0,\ \xi \in (-\infty, -1) \cup (1,\ \infty)$ and $V(\xi) = \frac{V_0}{e},\ \xi \in (-1,\ 1)$. In this case we have a constant speed $\frac{V_0}{e} < V_0$ with discontinuities

(27) at A, B borders (Fig. 1).

Because AB is curvilinear, in O the speed should be maximal and the profile is near the segment AB.

That is why, to create a depressure and to obtain the maximal lift we consider a speed distribution without discontinuities at A, B, and $V(\xi) \leq V^0,\ V(\pm 1) = V^0$,

$$V(\xi) = V^0\frac{1}{1+a\sqrt{1-\xi^2}},\ \xi \in (-1,\ 1),\ a \geq 0$$

$$t = \ln\frac{V^0}{V} = \ln\left(1 + a\sqrt{1-\xi^2}\right) \qquad (29)$$

If we know $V^0,\ 2L = \overline{AB}$, is possible to compute the parameter a, $2l = AB$ and the parameter k for obtaining the optimality. The relationships and the rationing are similar those presented in Section 3, for a condition of optimality: $t = t(\xi, a^*)$, $\theta = \theta(\xi, a^*)$, with $a = a^*$ corresponding to the maximal lift $C_y(a = a^*)$.

In this case

$$2L = A\int_{-1}^{1} e^{t(s)}ds = A\left\{2 + \frac{a\pi}{2}\right\},\ A = \frac{4L}{4 + a\pi} \quad (30)$$

The lift $Y = \rho V^{0^2} A\int_{-1}^{1} t(s)ds,\ 1 + k = \frac{L}{l}$ and the lift coefficient are:

$$C_y = \frac{Y}{\rho V^{0^2} l},\ C_y(a) = I(t(a))(1 + k),$$

$$I(a) = 2\frac{\int_{-1}^{1} t(s)ds}{\int_{-1}^{1} e^{t(s)}ds}$$

(31)

The optimization is done according $I(a) = \frac{2I_1(a)}{I_2(a)}$, and the integrals $I_1(a), I_2(a)$ are computed for $a \in (0, 1]$ and $a \in (1, \infty)$.

1[st] case: For $a \in (0,1)$ we have:

$$I_1 = \frac{\pi - 2a - 2\sqrt{1 - a^2}\arccos(a)}{a},$$

$$I_2 = 2 + \frac{a\pi}{2},\ a \in (0,1] \quad (32)$$

It results $I = I(a),\ I'(a) > 0$ upward and,

$$C_y(\max) = C_y(a_1^* = 1)$$

$$C_y(\max) = \frac{4(\pi - 2)}{\pi + 4}(1 + k) = 0.639405(1 + k) \quad (33)$$

In addition $(1 + k) = \frac{L}{l}$ is done according

$$\frac{1}{L} = \int_{-1}^{1} \cos\theta(\xi, a^*)d\xi,$$

where:

$$\theta(\xi, a = a^*) = -\frac{1}{\pi}\int_{-1}^{\xi} \frac{t(s, a^*)}{s - \xi}ds,\ \xi \in (-1,1) \quad (34)$$

2[nd] case: When a > 1, the maxim of the function $I = I(a)$:

$$I_1(a) = \frac{\pi - 2a - \sqrt{a^2 - 1}\ln\left[-1 + 2a\sqrt{a^2 - 1}\right]}{a},$$

$$I_2 = \frac{4 + a\pi}{2} \quad (35)$$

From $I = I(a)$, $I'(a^*) = 0,\ I(a)$ it results a maximum in $a_2^* = 2.16393$ and, because $I(a)$ is continue in a = 1, for $a \in (0, \infty)$ the global maximum will be obtained for $a^* = 2.16393$, or $I = I(a^*)$ and

$$C_y(\max) = 0.72122(1 + k) \quad (36)$$

The numerical computation for (1+k) will be made for this value, a^* and the optimal geometric shape is:

$$X = \frac{x(a^*, \xi)}{L},\ Y = \frac{y(a^*, \xi)}{L},\ \xi \in (-1,1)$$

The numerical result is:

k+1=1.19206 and $C_y(\max) = 0.86948$, (37)

very near the previous 0.86468 [2], and corresponds to:

$$\frac{V^0}{e} \approx \frac{V^0}{1 + a^*\sqrt{1 - \xi^2}},$$

$$a^* \approx 2.18878 \quad (38)$$

5. CONCLUDING REMARKS

The inverse method and the integral singular equations presented here is a general one, and permits to determine the geometric design of aerodynamic airfoils in an exact manner.

The solutions that represents the distribution of speeds, angles or pressures on the determined profile are also cases that optimizes the aerodynamic forces: the maximal drag or the maximal lift.

The optimization is proceeed by extreming the non- linear integral operators, obtaining the analytic exact solutions. The applications are important in the aerodynamics of low speed small UAVs [9,10] and these results could be extended to the case of compressible subsonic regime, [11] or the case of axysimmetric shapes.

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